**Principles of Communication Systems**

**Columbia University**

**ELEN E3701**

**Spring Semester- 2025**

**Problem Set # 12**

**Problem Set Due: 1 May 2025**

Problem # 1

In class we discussed the equation for the capacity, C, for a general channel with transfer function equal to H(f) and additive white gaussian noise, N0/2 watts/Hz.

The equation is shown below.

**∞**

**C=∫ df log2 1+⏐Hc(f) ⏐2 Ps(f) bps**

**0 N0/2**

**We also know that the total area under the power spectral density, Ps(f), of the transmitted signal, is equal to the transmitted power, P.**

**∞**

**∫Ps(f) df=P**

**-∞**

It was mentioned in class that the power spectral density, Ps,opt(f), which maximizes the capacity, C, is given by the equation below.

**λ - N0/2 ; f ε Fopt**

**Hc(f) 2**

**Ps opt(f) = f ε Fopt ; if Ps opt(f) ≥ 0**

**0 otherwise**

Fopt is the range of frequencies in which Ps,opt(f) exists (> 0).

If we place the optimum power spectral density, Ps,opt(f), in the equation for capacity, show that the maximum value of the capacity is given by the equation below.

**Copt=∫ df log2  λ ⏐Hc(f) ⏐2 bps**

**fεFopt N0/2**

Ps,opt(f) is called the “water-pouring” or “water-filling” solution for the problem of maximizing the capacity.